GAME AND STRATEGIES–Exam, January 22, 2019

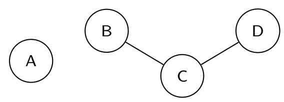
Surname: Name: Matricola:

Points:
Ex. 1: /9 **Ex. 2**: /9 **Ex. 3**: /8 **Ex. 4**: /8.

SOLVE THE EXERCISES AND ANSWER THE QUESTIONS USING ONLY THESE PAPERS

Exercise 1

A guard is trying to catch a burglar in a museum with 4 rooms connected as in the map below. The guard will catch the burglar if they happen to be either in the same room or in contiguous room. In that case the guard gets a payoff of 1 and the burglar -1. Otherwise the guard gets -1 and the burglar 1.



1. Say if there are weakly/strictly dominated strategies for the guard and for the burglar
2. Formulate this as a zero sum game and write the matrix of the game

1. B and D are weakly dominated by C for the guard (third row weakly dominates second and fourth row), C is weakly dominated for the burglar (by B or D).
2. It is possible to assign 1 to the guard if she captures the burglar and -1 if she fails, and conversely -1 to the burglar if captured, 1 otherwise. This is clearly a zero sum game, whose matrix is given by

$$\begin{pmatrix} 1 & -1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{pmatrix}$$

   where it appears that the third row dominated the second and the fourth one, while the third column is dominated by the second one

Exercise 2

Given the following zero sum game

$$\begin{pmatrix} 3 & 2 & 5 \\ 3 & 4 & 4 \\ 4 & 1 & 1 \end{pmatrix}.$$

1. Find the conservative values of the players in pure strategies
2. Reduce the matrix eliminating weakly dominated strategies
3. Solve the reduced game

1. $v_1 = 3, v_2 = 4$, no equilibrium in pure strategies;
2. by eliminating weakly dominated strategies the matrix reduces to

$$\begin{pmatrix} 3 & 4 \\ 4 & 1 \end{pmatrix}.$$

3. applying the indifference principle we get $(\frac{3}{4}, \frac{1}{4})$ are optimal for both players.

Exercise 3

A new freeway connecting two important cities is proposed. To decide whether it is worth for the community to built it, the state decides for a VCG mechanism: the citizens of the two cities are asked how do they value the presence of the new freeway. Assume that the number of citizens is 10^5 and that the capital needed to for the construction of the freeway is $c = 10^8$.

1. Describe what would be the outcome in the following scenarios:
   1. All the citizens report that they will get a benefit of 900;
   2. 50% the citizens report that they will get a benefit of 900, while 50% the citizens reports a benefit of 1200;
   3. 99.000 evaluate 990, 1000 evaluate 1991.

1. Since $10^5 \times 900 < 10^8$, the bridge will not be built;
2. Since $5 \times 10^4(900 + 1200) = 102 \times 10^6 > 10^8$ the bridge will be built. No citizen is pivotal, so nobody will pay anything;
3. The total evaluation is $100.001.000$ thus the bridge will be built. Citizens evaluating 900 are not pivotal, they pay nothing, citizens evaluating 1991 are pivotal, they pay 991.

Exercise 4

Given the weighted majority game $[24; 12, 13, 3, 5]$

1. Enumerate the winning coalitions
2. Say if there are symmetric players
3. Say if there are veto players and find the core of the game
4. Find the Shapley value of the game

1. The minimal winning coalition is $\{1, 2\}$, thus the winning coalitions are the coalitions A such that $\{1, 2\} \subseteq A$.
2. Players 1 and 2 are symmetric since they are Veto players, players 3 and 4 are symmetric, since they are Null players
3. The core is the set $C(v) = \{(x, 1 - x, 0, 0) : 0 \leq x \leq 1\}$.
4. The Shapley value is $(\frac{1}{2}, \frac{1}{2}, 0, 0)$.